What is a Logic?

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PHILOSOPHICAL, LOGICAL, AND EXPERIMENTAL ROUTES TO SUBSTRUCTURALITY

- The main proposal of this talk is to analyze **what is a logic.**
- I am going to present some ideas connecting **non-classical** and **sub-structural logics**

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- I am going to argue against the naive view according to which a logic is (only) a set of validities.
- I will explore logics in which the sets of the validities and metavalidities do not coincide.
- a non-classical classical logic
- a meta-classical non-classical logic.

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Step 1: Logic is not a particular presentation

- A logic can be presented in different ways

CL	LK	/	Boolean Models / SK models
IL	LI	/	Kripke Models
LP	Tableaux	/	SK Models

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What is a good criterion for the identity of a logic?

- A same logic can be presented with different calculus

- Hilbert calculus / Natural deduction

Then, a logic is not the calculus

What is a good criterion for the identity of a logic?

- A same logic can be presented with different models

For ejemple, CL can be presented using

2-valued logic + LC

SK + ST or WK + ST …
SK + TS/ST ….

- Step 2: a logic cannot be (only) a set of validities

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- A logic is a theory of the logical consequences

- Then, a natural way is to identify a logic with a set of valid inferences.

Theory of proof / Theory of models

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However…

CL and LP coincide in the set of tautology: for example, both validates sMP. If a logic is only a set of logical truths, CL and LP are the same logic. Of course, this is absurd !

As well a logic cannot be a set of logical truths, a logic cannot be a set of inferences.

However…

Barrio et al. [2015] have emphasized that ST is not classical logic, precisely because the absence of Cut makes classical logic and ST different regarding the metainferences they determine as valid.

So, Barrio et al. [2020] introduced the logic TS/ST, whose consequence relation is defined for metainferences, and so is a metainferential logic. TS/ST is characterized through the non-transitive logic ST and the non-reflexive logic TS.

The wrong way…

If a logic is (only) a set of logical truths , then LP and CL are the same logic.

If a logic is (only) a set of validities, then ST and CL are the same logic.

The extensional notion of what is a logic cannot be right

A logic cannot be a set of first-level inferences and metainferences either.

While ST coincides with classical logic in each valid inference (but not in every classically valid metainference), TS/ST recovers every classically valid metainference.

However, it fails to validate some classically valid meta-metainferences (metainferences of level 2). Thus, a new logic for meta-metainferences based on TS/ST can be defined to recover them. But then again, it fails to validate classically valid metainferences at higher levels (metainferences of level n > 2).

The Family of Strict-Tolerant logics

S T

SS TS ST TT

SS/SS TS/SS ST/SS TT/SS SS/TS TS/TS ST/TS TT/TS SS/ST **TS/ST** ST/ST TT/ST SS/TT TS/TT ST/TT TT/TT

…….. **ST/TS//TS/ST ……**

Barrio et al. [2020] introduced a hierarchy of sets of validities…

- **Step 3: a logic cannot be (only) a set of meta … meta validities.**

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Scambler (2020) shows that the hierarchy is not enough

Logics with same sets of inferences and metainferences valid can be different: the set of **anti-validities** can not coincide.

Barrio & Pailos (2022) shows how to add sets of anti-validities (in all metainferences levels.

Barrio & Pailos (2022) also shows that contingencies can be import

Then, we will develop a multi-standard approach to elaborate a new logic that captures not only every classical validity, but also every classical antivalidity and contingency.

Is this enough?

A logic is a theory of what inference and metainferences we have to accept and reject.

Of course, as CL, some theories has a homogeneous policy: they are classical or paracomplete in all levels and some are not. It's the same with the sets of anti-validities.

For same logics the logic of inference and metainferences coincide.

This is not the case for substructural logics: ST for example validates HS but rejects transitivity as metaentailment.

- **Step 4: Metainferential logics capture the logical of the entailments**

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Some metainferences codify structural properties of the logical consequence of a logic

As is well known, the deduction theorem links logical implication to the conditional. That is, it relates implication to the conditional of the language, allowing for an internal mode of representation.

When TD fails, logics with mixed policies can be developed. And as we have seen with the ST-hierarchy, we can represent the structural failures of implication in internal logic.

For example, we can internalize cutting failures as failures of the hypothetical syllogism.


Metainferential logics are theories about the logical links between inferences. They take inferences as objects and tell us which inferences are valid between those objects.


What is a substructural logic?

In the traditional sense, ST is not a substructural logic: its presentation in the calculus of sequents has no explicit cut but its notion of consequence is Tarskian. At its core, ST is LP. And LP has a Tarskian notion of consequence.

By making metainferences constitutive of logic, substructurality ceases to be a matter of presentation of the calculus.

And substructurality, when represented with meta-connectives, can be studied in all its logical features.


The idea is that just as paraconsistent logics are not necessarily inconsistent systems, but systems that can be non-trivially extended to inconsistent theories,

Substructural logics are not necessarily systems (sets of inferences) that are not closed under Tarskian properties, but systems that can be non-trivially extended to non-Tarskian consequence relations.

Classical logic is trivial in the presence of vague expressions or semantic predicates. One way to avoid triviality by extending its language with such expressions is to weaken the Tarskian properties of its notion of consequence. With ST substructure logic we do exactly that. And by turning the pure relations between its inferences into logic we understand the logical behavior of its external notion of consequence.

If we do not do meta-inferential logic, we do not fully understand the logical behavior of its implication.

- Step 5: There are metainferences in our practice

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We make meta inferences and we need to know what is valid

There are some arguments that do not involve any logical connective: “p therefore p”, “p, q, therefore q”, if we allow multiple conclusions also “p, q, therefore p, q”, and so on.

The validity or otherwise of these arguments does not depend on the meaning of the logical connectives; rather, it is determined by the meaning of logical consequence and, in particular, by the structural properties that logical consequence displays.

Metainferences are part of our reasoning

We make inferences between inferences: typically when proving theorems we use corollaries or concatenate sequences of arguments in our discussions.

Metainferences help us express pure features of the consequence relation.

Determining what is valid to meta-infer and what should not is part of the logic.

Conclusions

- The naive way to define what is a logic is wrong
- The correct way to characterize what a logic is must take into account which metainferences are valid and what a logic rejects. This allows us to establish what we can and cannot metainfer when we take into account the links between inferences.
- All logic as a theory must be closed under reflexivity, monotony and transitivity. But, internally we can represent with connectives, failures in these properties. Substructural logics are non-classical logics that internalize these failures.

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Thanks