Pattern Models and Action Models are Incomparable in Update Expressivity

Armando Castañeda
Hans van Ditmarsch
David A. Rosenblueth
Diego A. Velázquez

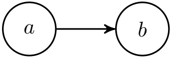
Institut de Recherche
en Informatique de Toulouse
CNRS - Toulouse INP - UT3 - UT Capitole - UT2

1. Context

Dynamic-Network Models

Communication is performed in synchronous rounds

Adversary

- A set of infinite sequences of communication graphs (Reflexive directed graphs)
    - 

Oblivious adversary

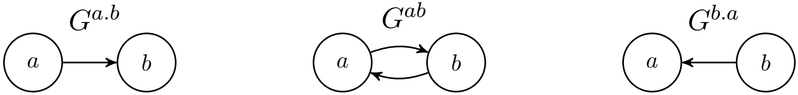
- Any communication graph in a given set X may occur in any round
- We say that X is the adversary

1. Context

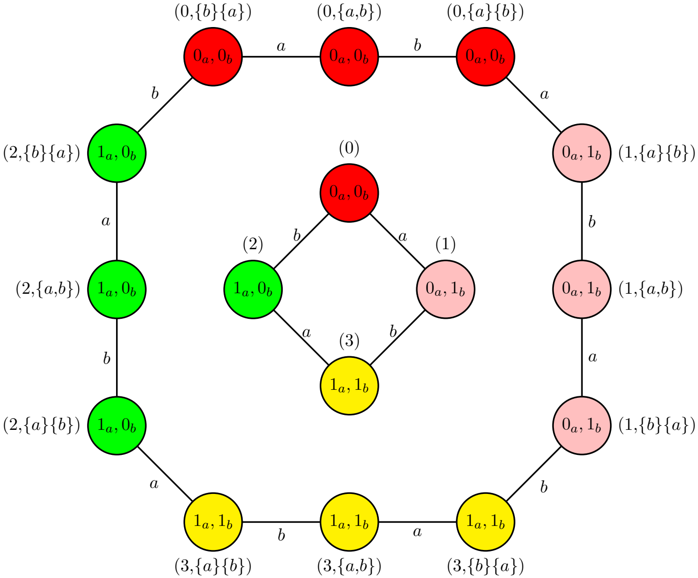
Iterated Immediate Snapshot (IIS)

- IIS can be described as an oblivious dynamic-network model

Two processes



1. Context
After first round



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- Dynamic Epistemic Logics (DEL)
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- Pattern Model Logic
  - Designed for analyzing distributed systems
    - Who communicates with whom? (communication graph)
    - full-information communication
    - A graph precondition depends on the model

2. Action Models and Pattern Models

Languages

Given a set of agents A and a set of propositions P,

- $\mathcal{L}_D$
  - $\phi := p_a \mid \neg\phi \mid \phi \wedge \phi \mid D_B\phi$
- $\mathcal{L}_\otimes$
  - $\phi := p_a \mid \neg\phi \mid \phi \wedge \phi \mid D_B\phi \mid [\mathsf{U}, \mathsf{e}]\phi$
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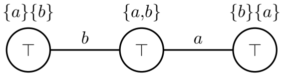
2. Action Models and Pattern Models
Action Model

$\mathsf{U} = (\mathsf{E}, \mathsf{R}, \mathsf{Pre})$

- E a set of events
- $\mathsf{R} : A \to \wp(\mathsf{E} \times \mathsf{E})$ **(indistinguishability)**
- $\mathsf{Pre} : \mathsf{E} \to \mathcal{L}_D$



2. Action Models and Pattern Models

Restricted Modal Product $\otimes$

$M' = (W', \sim', L') = M \otimes \mathsf{U}$

- $W' = \{(w, \mathsf{e}) \in W \times \mathsf{E} \mid M, w \models \mathsf{Pre}(\mathsf{e})\}$
- $\sim'_a = \{((w, \mathsf{e}), (w', \mathsf{e}')) \in W' \times W' \mid w \sim_a w' \ \wedge \ \mathsf{e}\ \mathsf{R}_a\ \mathsf{e}'\}$
- $L'((w, \mathsf{e})) = L(w)$

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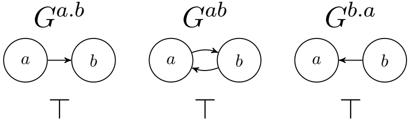
Pattern Model

$\mathcal{P} = (\mathbf{G}, Pre)$

- $\mathbf{G}$ a set of communication graphs
- $Pre : \mathbf{G} \to \mathcal{L}_D$



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Ga in-neighborhood of a in G

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Semantics on epistemic models

- $M, w \models p_a$ iff $p_a \in L(w)$
- $M, w \models \neg\phi$ iff $M, w \not\models \phi$
- $M, w \models \phi \wedge \psi$ iff $M, w \models \phi$ and $M, w \models \psi$
- $M, w \models D_B\phi$ iff $M, v \models \phi$ for all $v \sim_B w$

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- $M, w \models [\mathcal{P}, G]\phi$ iff $M, w \models$ $Pre(G)$ implies $M \odot \mathcal{P}, (w, G) \models \phi$

2. Update expressivity

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By notational abbreviation,

- $[\mathsf{U}]\phi := \bigwedge_{\mathsf{e}\in E}[\mathsf{U},\mathsf{e}]\phi$
- $[\mathcal{P}]\phi := \bigwedge_{G\in\mathbf{G}}[\mathcal{P},G]\phi$

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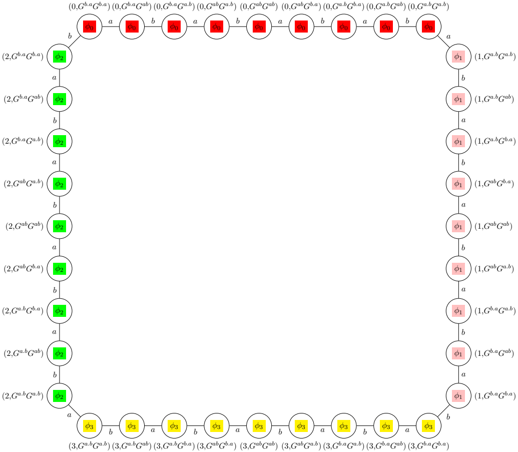


update effect as U?

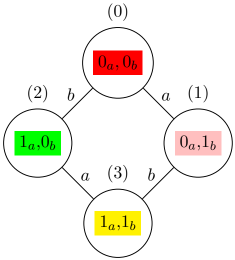
2. Update expressivity

$\phi_0 = 0_a \wedge 0_b$

$\phi_1 = 0_a \wedge 1_b$

$\phi_3 = 1_a \wedge 1_b$

$\phi_2 = 1_a \wedge 0_b$



M^0

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Pattern models are not at least as update expressive as action models

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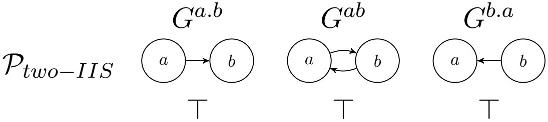

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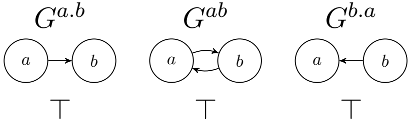
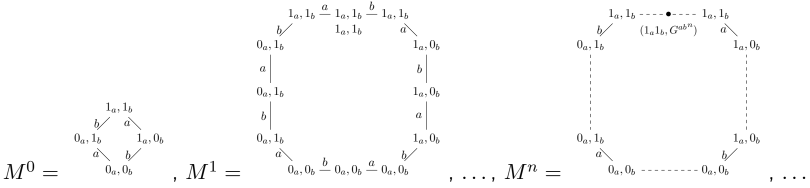
$\mathcal{P}_{two-IIS}$



- $M^0(\odot\mathcal{P}_{two-IIS})^n$

2. Update expressivity



$$(\bullet): \quad (3, G^{ab^{n-1}} G^{b.a}) \overset{a}{\text{———}} (3, G^{ab^n}) \overset{b}{\text{———}} (3, G^{ab^{n-1}} G^{a.b})$$

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$$n > \log_3 2(md(\mathsf{U}) + 1)$$

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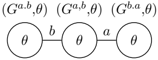

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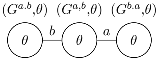

$$\begin{tikzcd}[column sep=small]
(\cdot,(G^{b.a},\theta)) \arrow[r, dash, "a"] \arrow[rrrrr, dash, bend left, "a"] & (\cdot,(G^{ab},\theta)) \arrow[r, dash, "b"] \arrow[rrr, dash, dashed, bend left, "a"] & (\cdot,(G^{a.b},\theta)) \arrow[r, dash, "a"] & (\cdot,(G^{a.b},\theta)) \arrow[r, dash, "b"] \arrow[rrrrr, dash, bend right, "b"'] & (\cdot,(G^{ab},\theta)) \arrow[r, dash, "a"] \arrow[rrr, dash, bend right, "b"'] & (\cdot,(G^{b.a},\theta)) \arrow[r, dash, "b"] & (\cdot,(G^{b.a},\theta)) \arrow[r, dash, "a"] & (\cdot,(G^{ab},\theta)) \arrow[r, dash, "b"] & (\cdot,(G^{a.b},\theta))
\end{tikzcd}$$

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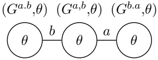

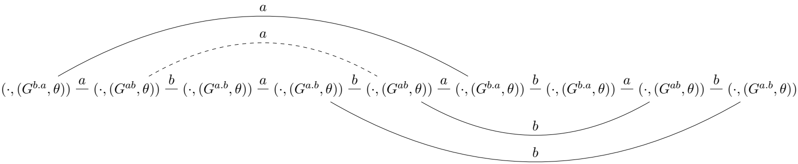

There is a shorter path to the worlds with different labeling above

2. Update expressivity

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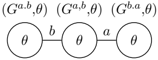

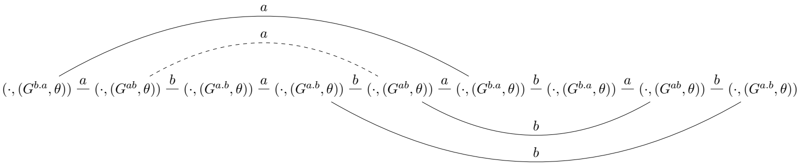

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Action models are not at least as update expressive as pattern models

2. Update expressivity

Action models and pattern models are incomparable in update expressivity


thank you
danke
謝謝
ngiyabonga
teşekkür ederim
gracias
merci
obrigado
dziękuję
bedankt
спасибо
Баярлалаа
рахмат
go raibh maith agat
mochchakkeram
sukriya
kop khun krap
terima kasih
grazie
arigatô
dakujem
takk
tapadh leat
ありがとう
감사합니다
ευχαριστώ
мерси
хвала
dank je
blagodaram
vinaka
mersi
barka
welalin
tack
spas
misaotra
matondo
paldies
grazzi
mahalo
asante
manana
obrigada
tenki
murakoze
chokrane
mamnun
дякую
tau
djiere dieuf
akun
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aciu
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gratias ago
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mauruuru
hvala
gràcie
bayarlalaa
dhanyavad
kiitos
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nandri
nanni
faafetai lava
спасибі
enkosi
sobodi
dėkuji
mèsi
didi madloba
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mercé
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