Genuinely non-traditional logics

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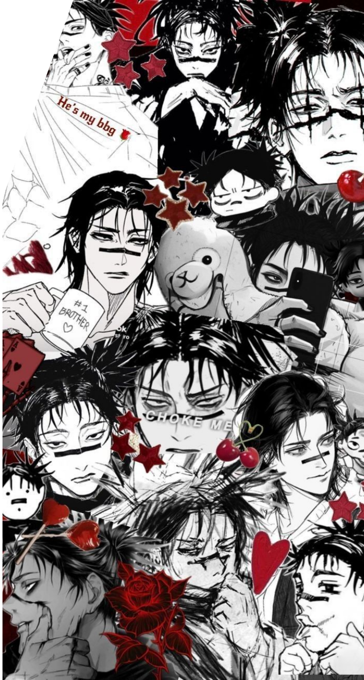
LANMR 2024

This work was supported by the PAPIIT project IG400422 and the CONAHCYT project CBF2023-2024-55.



Objective

I want to answer the following question:

Is there any genuinely non-traditional logic?

Genuine properties

1 Introduction

Béziau and Franceschetto: **L** is a *genuinely paraconsistent logic* iff:

$$\text{(GPcons1)} \not\models_{\mathbf{L}} N(A \otimes NA)$$
$$\text{(GPcons2)}\ A \otimes NA \not\models_{\mathbf{L}}$$

where N is some negation and $\otimes$ is some conjunction.

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Genuine properties

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Genuine properties

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L3 and **K3** satisfice the condition (GPcons1).
LP satisfice the condition (GPcons2).
L3A and **L3B** two extensions of logic **C1** satisfies the conditions (GPcons1) and (GPcons2).

Genuine properties

Tello, Borja and Coniglio: **L** is a *genuinely paracomplete logic* iff:

$$\text{(GPcomp1)} \not\models_{\mathbf{L}} (A \oplus NA)$$
$$\text{(GPcomp2)}\ N(A \oplus NA) \not\models_{\mathbf{L}}$$

where N is some negation and $\oplus$ is some disjunction.

Genuine properties

Example:

Example:

IPL satisfies the condition (GPcomp1).

Genuine properties

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IPL satisfies the condition (GPcomp1).
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Genuine properties

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IPL satisfies the condition (GPcomp1).
LP satisfies the condition(GPcomp2).
L3AD and **L3BD**satisfies the condition (GPcomp1) and (GPcomp2).

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Genuine properties

- A logic **L** is *genuinely paranormal* if and only if it satisfies the following conditions:

$$\text{(GPcons1)} \nvDash_{\mathbf{L}} N(A \otimes NA)$$
$$\text{(GPcons2)}\ A \otimes NA \nvDash_{\mathbf{L}}$$
$$\text{(GPcomp1)} \nvDash_{\mathbf{L}} (A \oplus NA)$$
$$\text{(GPcomp2)}\ N(A \oplus NA) \nvDash_{\mathbf{L}}$$

That is, whether it is genuinely paraconsistent and genuinely paracomplete.

Example:

Genuine properties

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FDE satisfies the conditions (GPcons 2) and (GPcomp1).

Genuine properties

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N4 satisfies the conditions (GPcons1), (GPcons2), (GPcomp1) and (GPcomp2).

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FDE satisfies the conditions (GPcons 2) and (GPcomp1).
N4 satisfies the conditions (GPcons1), (GPcons2), (GPcomp1) and (GPcomp2).
FDE satisfies the conditions (GPcons1), (GPcons2), (GPcomp1) and (GPcomp2).

Genuine properties

L is a *genuinely non-reflexive logic* if and only if:

$$\text{(GNR1)} \not\models_{\mathbf{L}} A > A$$
$$\text{(GNR2)}\ N(A > A) \not\models_{\mathbf{L}}$$

where N is some negation and $>$ is some conditional.

Genuine properties

The law of Identity “A is A‘’ is not about the conditional, but about equality.

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In the history of logic sentences like “A is B‘’ were treated as:

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In the history of logic sentences like “A is B‘’ were treated as:

Preaching by the Stoics and the Peripatetics.

Conditional by Leibniz.

Genuine properties

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Example:

K3 satisfies the condition (GNR1).

Genuine properties

Example:

K3 satisfies the condition (GNR1).

M3V satisfies the condition (GNR2).

Example:

K3 satisfies the condition (GNR1).

M3V satisfies the condition (GNR2).

Genuine properties

- A logic **L** is genuinely *non-traditional* if and only if it satisfies the following conditions:

$$\text{(GPcons1)} \not\models_{\mathbf{L}} N(A \otimes NA)$$
$$\text{(GPcons2)}\ A \otimes NA \not\models_{\mathbf{L}}$$
$$\text{(GPcomp1)} \not\models_{\mathbf{L}} (A \oplus NA)$$
$$\text{(GPcomp2)}\ N(A \oplus NA) \not\models_{\mathbf{L}}$$
$$\text{(GNR1)} \not\models_{\mathbf{L}} A > A$$
$$\text{(GNR2)}\ N(A > A) \not\models_{\mathbf{L}}$$

That is, if it is genuinely paranormal and genuinely non-reflective.

Is there any genuinely non-traditional logic?

Genuine properties

Is there any genuinely non-traditional logic?

Yes, **FDE**!

Outline

- Introduction
- **FDE**
- **FDE** and the Genuine properties
- Problems with FDE as well as some solutions
- Genuine non-reflexivity and logical consequence
- Conclusions

FDE

Our formal lenguaje $\mathcal{L}$ consists of formulas constructed, in the usual way as follows:

$$A ::= p| \sim A|A \wedge A|A \vee A|$$

- A *interpretation* of $\mathcal{L}$ is a function $\sigma : Var \longrightarrow \{\{1\}, \{1, 0\}, \{\ \}, \{0\}\}$ from the set of propositional variables to the set $\{\{1\}, \{1, 0\}, \{\ \}, \{0\}\}$, where ‘1’ and ‘0’ mean true and false, respectively.

- In **FDE** the connectives are interpreted as follows.

$1 \in \sigma(\sim A)$ iff $0 \in \sigma(A)$
$0 \in \sigma(\sim A)$ iff $1 \in \sigma(A)$

$1 \in \sigma(A \wedge B)$ iff $1 \in \sigma(A)$ and $1 \in \sigma(B)$
$0 \in \sigma(A \wedge B)$ iff $0 \in \sigma(A)$ or $0 \in \sigma(B)$

$1 \in \sigma(A \vee B)$ iff $1 \in \sigma(A)$ or $1 \in \sigma(B)$
$0 \in \sigma(A \vee B)$ iff $0 \in \sigma(A)$ and $0 \in \sigma(B)$

FDE

The above model-theoretic semantics for **FDE** can be represented in tabular form as follows:

A	$\sim A$
$\{1\}$	$\{0\}$
$\{1,0\}$	$\{1,0\}$
$\{\ \}$	$\{\ \}$
$\{0\}$	$\{1\}$

$A \wedge B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1,0\}$	$\{1,0\}$	$\{0\}$	$\{0\}$
$\{\ \}$	$\{\ \}$	$\{0\}$	$\{\ \}$	$\{0\}$
$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$	$\{0\}$

$A \vee B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{1\}$	$\{1,0\}$
$\{\ \}$	$\{1\}$	$\{1\}$	$\{\ \}$	$\{\ \}$
$\{0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$

FDE

- A is a logical consequence Γ (in **FDE**), $\Gamma \models_{\textbf{FDE}} A$, if and only if, for every evaluation σ, if $1 \in \sigma(B)$ for every $B \in \Gamma$, $1 \in \sigma(A)$.

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- A is a emphlogical truth in L if and only if $\Gamma \models_{\textbf{FDE}} A$ and $\Gamma = \varnothing$.
- An argument is *invalid* in **L** if and only if there exists an evaluation in which the premises are true, i.e., $1 \in \sigma(B)$ for every $B \in \Gamma$, but the conclusion is not true, i.e., $1 \notin \sigma(A)$.

FDE and genuine paranormality

In **FDE** the conditions (GPconsis1), (GPconsis2), (GPcomp1) and (GPcomp2) are satisfied.

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- (GPcons1) $\not\models_{\mathbf{L}} \sim (A \wedge \sim A)$ y (GPcomp1) $\not\models_{\mathbf{L}} (A \vee \sim A)$
  Let's consider the case where $\sigma(A) = \{\ \}$.

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  Let's consider the case where $\sigma(A) = \{\ \}$.
- (GPcomp2) $\sim (A \vee \sim A) \not\models_{\mathbf{FDE}}$ and (GPcons2) $A \wedge \sim A \not\models_{\mathbf{FDE}}$
  Let's consider the case where $\sigma(A) = \{1, 0\}$.

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3 FDE and the Genuine properties

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  Let's consider the case where $\sigma(A) = \{\ \}$.
- (GPcomp2) $\sim (A \vee \sim A) \not\models_{\mathbf{FDE}}$ and (GPcons2) $A \wedge \sim A \not\models_{\mathbf{FDE}}$
  Let's consider the case where $\sigma(A) = \{1, 0\}$.

So **FDE** is a genuinely paranormal logic.

What about genuine nonreflexivity?

FDE and genuine non-reflexivity

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In **FDE** an extensional conditional can be defined as $A \rightarrow B$ as $\sim A \vee B$. The evaluation conditions for that connective are as follows:

$1 \in \sigma(A \rightarrow B)$ iff $0 \in \sigma(A)$ or $1 \in \sigma(B)$

$0 \in \sigma(A \rightarrow B)$ iff $1 \in \sigma(A)$ and $0 \in \sigma(B)$

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$0 \in \sigma(A \to B)$ iff $1 \in \sigma(A)$ and $0 \in \sigma(B)$

And its tabular representation is as follows:

$A \to B$	$\{1\}$	$\{1, 0\}$	$\{\,\}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1, 0\}$	$\{\,\}$	$\{0\}$
$\{1, 0\}$	$\{1\}$	$\{1, 0\}$	$\{1\}$	$\{1, 0\}$
$\{\,\}$	$\{1\}$	$\{1\}$	$\{\,\}$	$\{\,\}$
$\{0\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

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- (GNR2) $\sim (A \rightarrow A) \not\models_{\textbf{FDE}}$
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So, **FDE** is a genuinely non-reflexive logic.

Yeeeeeeeeei! **FDE** is a genuinely non-traditional logic.

Someone might object that $A \to B$ is not a conditional since it does not satisfy any of the following properties:
(Identity) $\models_{\mathsf{L}} A > A$
(Separation) $A, A > B \models_{\mathsf{L}} B$

Expansions of FDE

There are expansions of **FDE** with conditionals satisfying some expected properties such as Identity and Separation.

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(Separación) $A, A > B \models_{\mathsf{L}} B$
(Non-trivial separation) $A > B \not\models_{\mathsf{L}} A$ ó $A > B \not\models_{\mathsf{L}} B$

Otherwise, only Separation could not distinguish $>$ from a conjunction.

We can obtain a genuinely non-reflexive conditional satisfying separation and non-trivial separation, from the material conditional.

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The evaluation conditions of the material conditional are as follows.

$1 \in \sigma(A \supset B)$ iff $1 \notin \sigma(A)$ or $1 \in \sigma(B)$

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$0 \in \sigma(A \supset B)$ iff $1 \in \sigma(A)$ and $0 \in \sigma(B)$
And its tabular representation is as follows:

$A \supset B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{\ \}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$
$\{0\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

To obtain a genuinely non-reflexive conditional it suffices to change one of the assignments of the diagonal. Suppose the conditional is evaluated as follows.

$A \supset_{nr} B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{\ \}$	$\{1\}$	$\{1\}$	$\{\ \}$	$\{1\}$
$\{0\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

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$\{1,0\}$	$\{1\}$	$\{1,0\}$	$\{\,\}$	$\{0\}$
$\{\,\}$	$\{1\}$	$\{1\}$	$\{\,\}$	$\{1\}$
$\{0\}$	$\{1\}$	$\{1\}$	$\{1\}$	$\{1\}$

Its evaluation conditions are as follows:

$1 \in \sigma(A \supset_{nr} B)$ iff $0 \in \sigma(A)$ and $1 \notin \sigma(A)$, or $1 \in \sigma(B)$, either $0 \in \sigma(B)$ but $1 \notin \sigma(A)$
$0 \in \sigma(A \supset_{nr} B)$ iff $1 \in \sigma(A)$ and $0 \in \sigma(B)$
The resulting logic is **FDE**$_{\supset_{nr}}$ and again, it is a genuinely non-traditional logic.

It is not necessary to start from the material conditional. People like Fjellstad claim that, any conditional $A >_d B$ that validates Separation must meet this requirement:
If $1 \in \sigma(A >_d B)$ and $1 \in \sigma(A)$ then $1 \in \sigma(B)$
This requirement is equivalent to
If $1 \notin \sigma(B)$ then $1 \notin \sigma(A >_d B)$ or $1 \notin \sigma(A)$
We can obtain any table of the form

$A >_d B$	$\{1\}$	$\{1,0\}$	$\{\ \}$	$\{0\}$
$\{1\}$			$\{\ \}/\{0\}$	$\{\ \}/\{0\}$
$\{1,0\}$			$\{\ \}/\{0\}$	$\{\ \}/\{0\}$
$\{\ \}$				
$\{0\}$				

Genuine non-reflexivity and logical consequence

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$\Gamma \models^{q}_{\mathsf{L}} A$ if for allσ, si $1 \in \sigma(B)$ or $0 \notin \sigma(B)$, for every $B \in \Sigma$, then $1 \in \sigma(A)$ and $0 \notin \sigma(A)$.

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Notion of logical consequence close to nonreflexivity, namely Malinowski's q consequence also called TS consequence.

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It is easy to verify that $A \not\models^{q}_{\mathsf{L}} A$, considering the cases in which $\sigma(A) = \{1, 0\}$ or $\sigma(A) = \{\,\}$.

With the consequence q, genuine nonreflexivity already appears without modifying the material conditional.

Genuine non-reflexivity and the logical consequence

With the consequence q, genuine nonreflexivity already appears without modifying the material conditional.

- (GNR1)$\not\models^{q}_{\mathbf{FDE}_{\supset}} A \supset A$
  Let's consider the case where $\sigma(A) = \{1, 0\}$ and $1 \notin \sigma(B)$

With the consequence q, genuine nonreflexivity already appears without modifying the material conditional.

- (GNR1) $\not\models^{q}_{\mathbf{FDE}_{\supset}} A \supset A$
  Let's consider the case where $\sigma(A) = \{1, 0\}$ and $1 \notin \sigma(B)$
- (GNR2) $\sim(A \supset A) \models^{q}_{\mathbf{FDE}_{\supset}} B$
  Consider again a σ such that $\sigma(A) = \{1, 0\}$ and $1 \notin \sigma(B)$.

Genuine non-reflexivity and the logical consequence

With the consequence q, genuine nonreflexivity already appears without modifying the material conditional.

- (GNR1)$\not\models^{q}_{\mathbf{FDE}_{\supset}} A \supset A$
  Let's consider the case where $\sigma(A) = \{1, 0\}$ and $1 \notin \sigma(B)$
- (GNR2) $\sim(A \supset A) \models^{q}_{\mathbf{FDE}_{\supset}} B$
  Consider again a σ such that $\sigma(A) = \{1, 0\}$ and $1 \notin \sigma(B)$.

So, $\mathbf{FDE}_{\supset}$ with q consequence is genuinely nonreflexive.

Conclusions

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Conclusions

- I introduced the notion of genuinely nontraditional logics.
- I showed that **FDE** is a genuinely nontraditional logic.
- The authenticity of the conditional of **FDE** can be questioned.
- The semantics of **FDE** can be expanded with a conditional whose authenticity is not questioned.
- To obtain genuine reflexivity we can move into the realm of logical consequence..

Sankyuuuu!

Bien jalapeños!