Deliberative Semantics for a Term Logic Tableaux

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- We usually say, rather informally, that a logic is relevant when it avoids some counterintuitive classical inferences such as *ex falso* ($(\phi \wedge \neg\phi) \to \psi$) and *verum ad* ($\phi \to (\psi \to \psi)$).

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- This characterization, nevertheless, will not suffice because even if it states what seems to be a necessary condition of relevance, it allows too much: *petitio principii* $(\phi \vdash \phi)$, for example, would still be relevant, yet not quite reasonable (cfr. [MM19]).

- We might add, then, that a logic is relevant when it requires the antecedent and the consequent of an implication to be relevantly related, on topic. However, although true, this second description will not do because *ex falso* and *verum ad* might still be understood as complying this definition.

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- Consider, for instance, that to say that from ϕ we can deduce ψ is to say that the content of ψ is a part of the content of ϕ, in which case both *ex falso* and *verum ad*—not to mention *petitio*—would still be relevant insofar as contradictions and tautologies may be parts—or topics—of any content (cfr. [Mor83]).

- Consequently, in an effort to clarify and precisely define the notion of relevance we might move forward and say that a logic is relevant when we stipulate, rather formally, a variable sharing principle in order to demand that no formula of the form $\phi \rightarrow \psi$ is valid if ϕ and ψ do not have at least one variable in common, that is to say, that no inference can be shown to be valid if the premises and the conclusion do not share at least one and *the same* variable (cfr. [ADB75]).

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- This third, semantical formulation is way more clear and precise and suggests an understading of relevance that can be further developed, in a syntactical fashion, in terms of use-in-a-proof. Hence a logic is called relevant if in every theorem of the form $\phi \rightarrow \psi$, ϕ is used to prove ψ by way of some adequate rule of inference (cfr. [ADB75]).

This is neat, but again, these semantical and syntactical depictions are kind of odd for they still allow the validity of unsettling inferences such as *petitio*.

- There is another way to approach relevance though—in all fairness, there are many other ways (cfr. [Wal03]). We can move backwards and recall that Aristotle once suggested that a *petitio principii* is a fallacy because it fails to account for a causal explanation since it depends upon assuming what has to be explained (*De Sophisticis Elenchis* 168b23-27).

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- For Aristotle, as we will see, it is a requirement of a legitimate inference that the conclusion *has to be different* from the premises (*Topics* 100a25-26, *De Sophisticis Elenchis* 165a1-2, *Prior Analytics* 24b19-20).

This last notion is interesting because it points towards a sense of relevance understood in terms of a causal relatedness that avoids *petitio*—not to mention *ex falso* and *verum ad*—by design, and fortunately, one logic arranged to capture this sense of relevance is, of course, the traditional, Aristotelian logic, namely syllogistic (hence syllogistic relevance).

But much to our chagrin, the cultivation of this logic, in spite of certain current efforts [Vea70, Eng96, KD04, AC12, Mos15, ES11, Eng17, Cor1], has been disparaged in various ways, especially since the early 20th century (and not without good reasons [de 64, FA73, Rus, Car30, Gea62]).

One of these efforts is Sommers and Englebretsen's Term Functor Logic [Som82, SE00, Eng87, Eng96, ES11], a term logic that recovers some important features of the traditional, Aristotelian logic; however, as we will see, it turns out that said logic does not preserve all of the Aristotelian properties a proper inference should have insofar as the class of theorems of Term Functor Logic includes some inferences that may be considered causally irrelevant (e.g. *ex falso*, *verum ad*, and *petitio*).

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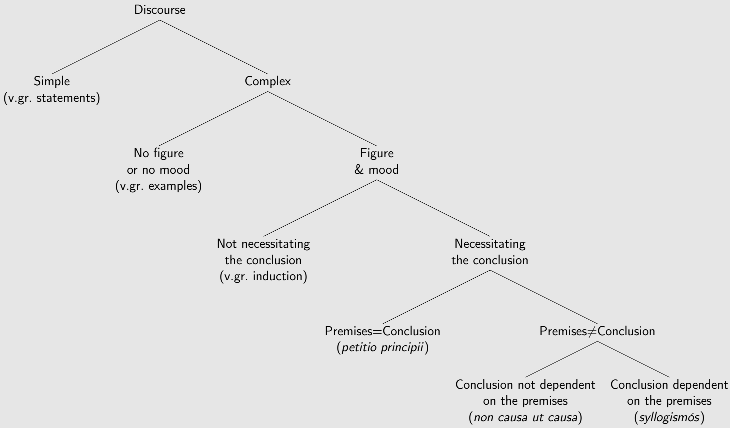
- Enter Sommers and Englebretsen's Term Functor Logic to solve the formal adequacy problem.
- However, irrelevance is parasitic of Term Functor Logic, thus compromising its material adequacy.
- Thus our contribution: offering some syntax/semantics to solve the irrelevance issue.

Syllogistic is a term logic that has its origins in Aristotle's *Prior Analytics* [Ari89] and deals with inference between categorical statements of the form:

$$\phi := \langle \mathit{Cuan} \rangle \mathsf{S} \langle \mathit{Cual} \rangle \mathsf{P}$$

where S and P are terms, *Cuan* may be *All* or *Some*, and *Cual* may be *is* or *is not*.

	Statement
1.	All computer scientists are animals.
2.	All logicians are computer scientists.
$\vdash$	All logicians are animals.


Discourse
Simple (v.gr. statements)
Complex
No figure or no mood (v.gr. examples)
Figure & mood
Not necessitating the conclusion (v.gr. induction)
Necessitating the conclusion
Premises=Conclusion (*petitio principii*)
Premises≠Conclusion
Conclusion not dependent on the premises (*non causa ut causa*)
Conclusion dependent on the premises (*syllogismós*)

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- By 1930, Carnap would generalize this judgment to all traditional logic by claiming that its available syntax was predicative only.
- Between 1960 and 1980, Geach would criticize term homogeneity.

Statement	Syllogistic	FOL
Every Greek is mortal.	GaM	$\forall x(Gx \Rightarrow Mx)$
Socrates is Greek.	SaG (?)	Gs
Socrates and Plato are friends.	?	Fsp
If you are Socrates, you are Plato's friend.	?	$S \Rightarrow P$
Every circle is a figure; thus, whoever draws a circle draws a figure.	?	$\forall x(Cx \Rightarrow Fx) \vdash \forall x((Dx \wedge \exists y(Cy \wedge Rxy)) \Rightarrow (Dx \wedge \exists y(Fy \wedge Rxy)))$

By contrast, genuine logic, namely first order logic, follows the Fregean paradigm that results:

- from dropping the term syntax and
- adopting a binary grammar of function-argument pairs.

This standard is familiar to us because we usually follow it when we teach, research, or apply logic: **this is the received view of logic**.

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- Term syntax.
- Term semantics.
- Rejection of *ex falso* and *verum ad*.
- Rejection of the collapse between the reflexivity of inference (i.e. $\phi \vdash \phi$) and the identity principle (i.e. $\phi \to \phi$)

Term Functor Logic (TFL, for short) [Som82, SE00, Eng87, Eng96, ES11] is a plus-minus algebra that employs terms and functors rather than first order language elements such as individual variables or quantifiers (cf. [Qui71, Noa80, Kuh83, Som82, Som05, Mos15]). According to this algebra, the four categorical statements can be represented by the following syntax [Eng96]:

- SaP $:= -S + P$
- SeP $:= -S - P$
- SiP $:= +S + P$
- SoP $:= +S - P$

Statement	Syllogistic	FOL	TFL
Every Greek is mortal.	GaM	$\forall x(Gx \Rightarrow Mx)$	$-\mathrm{G} + \mathrm{M}$
Socrates is Greek.	SaG (?)	Gs	$\pm\mathrm{s} + \mathrm{G}$
Socrates and Plato are friends.	?	Fsp	$\pm\mathrm{s} + \mathrm{F} + \pm\mathrm{p}$
If you are Socrates, you are Plato's friend.	?	$S \Rightarrow P$	$-[\mathrm{S}] + [\mathrm{P}]$
Every circle is a figure; thus, whoever draws a circle draws a figure.	?	$\forall x(Cx \Rightarrow Fx)$ $\vdash \forall x((Dx \land \exists y(Cy \land Rxy)) \Rightarrow (Dx \land \exists y(Fy \land Rxy)))$	$-\mathrm{C} + \mathrm{F}$ $\vdash -(+D + C) + (+D + F)$

$$\Gamma \vdash \phi \textit{ iff } \begin{cases} (i) \ \sum_{alg}(\Gamma) = \sum_{alg}(\phi), \textit{ and} \\ (ii) \ |particular(\Gamma)| = |particular(\phi)| \end{cases}$$

$$\Gamma \models \phi \textit{ iff } \sum_{arit}(\Gamma) = \sum_{arit}(\phi)$$

	Statement	TFL
1.	All computer scientists are animals.	$-C + A$
2.	All logicians are computer scientists.	$-L + C$
$\vdash$	All logicians are animals.	$-L + A$

As exposed in [CMRC18, CM20] and following [DGHP99, Pri08], we can develop a tableaux proof method for TFL.

$$\begin{array}{c} -A \pm B \\ \diagup \quad \diagdown \\ -A^i \qquad \pm B^i \end{array}$$

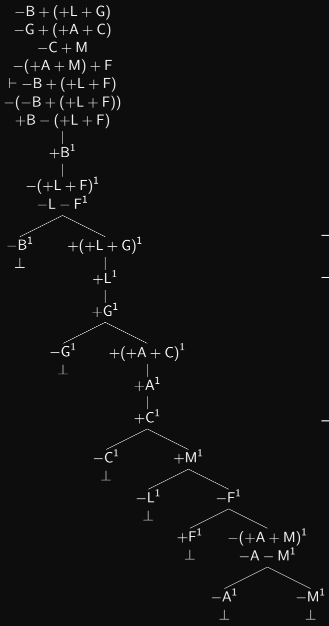
$$\begin{array}{c} +A \pm B \\ | \\ +A^i \\ | \\ \pm B^i \end{array}$$

$-M + P$
$-S + M$
$\vdash -S + P$
$-(-S + P)$
$+S - P$
$+S^1$
$-P^1$
$-S^1$ $\bot$ | $+M^1$
$-M^1$ $\bot$ | $+P^1$ $\bot$

$-M - P$
$-S + M$
$\vdash -S - P$
$-(-S - P)$
$+S + P$
$+S^1$
$+P^1$
$-S^1$ $\bot$ | $+M^1$
$-M^1$ $\bot$ | $-P^1$ $\bot$

$-M + P$
$+S + M$
$\vdash +S + P$
$-(+S + P)$
$-S - P$
$+S^1$
$+M^1$
$-M^1$ $\bot$ | $+P^1$
$-S^1$ $\bot$ | $-P^1$ $\bot$

$-M - P$
$+S + M$
$\vdash +S - P$
$-(+S - P)$
$-S + P$
$+S^1$
$+M^1$
$-M^1$ $\bot$ | $-P^1$
$-S^1$ $\bot$ | $+P^1$ $\bot$



	Statement
1.	Every B loves some G.
2.	Every G adores some C.
3.	Every C is M.
4.	Whoever adores something M is F.
⊢	Every B loves some F.

ex falso	*verum ad*	*petitio*
1. $-A + B$	1. $\pm B$	1. $\pm A$
2. $+A - B$	$\vdash -A + A$	$\vdash \pm A$
$\vdash -A + A$		

$$-A \pm B_f$$

$$-A^i_f \qquad \pm B^i_f$$

$$+A \pm B_f$$

$$+A^i_f$$

$$\pm B^i_{f'}$$

- Now, given any tableau, we say a branch is *open* if and only if there are no terms of the form $\pm A^i_f$ and $\mp A^i_f$ on it, for whatever flags; otherwise, it is not-open.

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- A not-open branch comes in various forms: a branch is *semi-open* (or *semi-closed*) if and only if there are terms of the form $\pm A^i_f$ and $\mp A^i_f$, for the same flag; otherwise it is *closed*.

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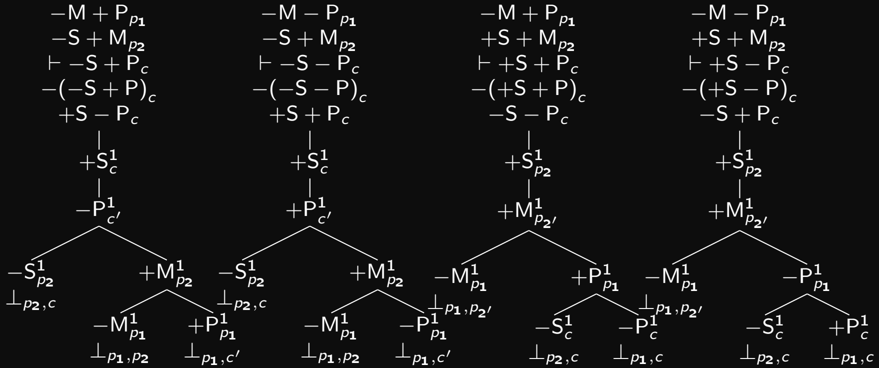
- An open branch is indicated by writing ∞ at the end of it; a semi-open (semi-closed) branch is indicated by writing $\propto_{f,f}$ ($\reflectbox{$\propto$}_{f,f}$); and a closed branch is denoted by $\perp_{f,f'}$.

- With these distinctions, we say a tableau is *Aristotelian* (or *propter quid*) if and only if every branch is closed and all the flags are carried at the end of every tip;
- a tableau is *open* (or *non sequitur*) if and only if it has an open branch;
- otherwise, it is *classical* (either *quia* or *non causa ut causa*).


$-M+P_{p_1}$
$-S+M_{p_2}$
$\vdash -S+P_c$
$-(-S+P)_c$
$+S-P_c$
$+S^1_c$
$-P^1_{c'}$
$-S^1_{p_2}$
$\bot_{p_2,c}$
$+M^1_{p_2}$
$-M^1_{p_1}$
$\bot_{p_1,p_2}$
$+P^1_{p_1}$
$\bot_{p_1,c'}$
$-M-P_{p_1}$
$-S+M_{p_2}$
$\vdash -S-P_c$
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$+S+P_c$
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$+P^1_{p_1}$
$-S^1_c$
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$-P^1_c$
$\bot_{p_1,c}$
$-M-P_{p_1}$
$+S+M_{p_2}$
$\vdash +S-P_c$
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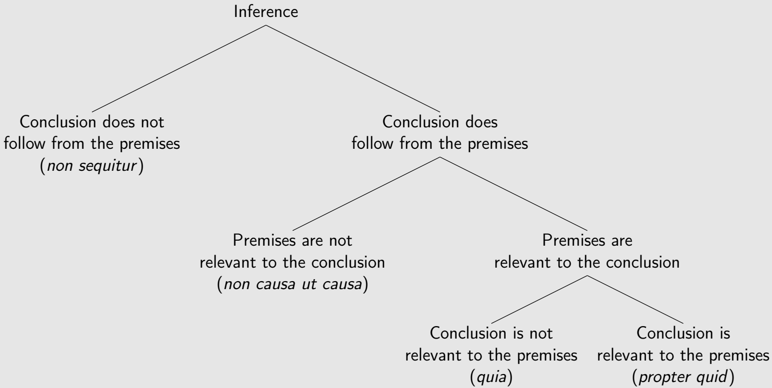
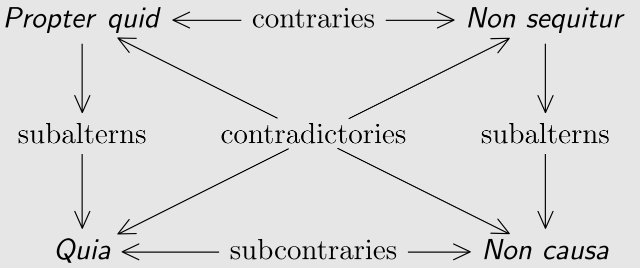
$-S+P_c$
$+S^1_{p_2}$
$+M^1_{p_{2'}}$
$-M^1_{p_1}$
$\bot_{p_1,p_{2'}}$
$-P^1_{p_1}$
$-S^1_c$
$\bot_{p_2,c}$
$+P^1_c$
$\bot_{p_1,c}$

$$\begin{array}{c}
\pm A_c \\
\vdash \pm A_c \\
-(\pm A)_c \\
\mp A_c \\
| \\
\pm A^1_c \\
| \\
\mp A^1_c \\
\asymp_{c,c}
\end{array}$$

$$\begin{array}{c}
\pm B_{p_1} \\
\vdash -A + A_c \\
-(-A + A)_c \\
+A - A_c \\
| \\
+A^1_c \\
| \\
-A^1_{c'} \\
\perp_{c,c'}
\end{array}$$

$$\begin{array}{c}
-A + B_{p_1} \\
+A - B_{p_2} \\
\vdash -A + A_c \\
-(-A + A)_c \\
+A - A_c \\
| \\
+A^1_c \\
| \\
-A^1_{c'} \\
| \\
+A^2_{p_2} \\
| \\
-B^2_{p_{2'}} \\
\swarrow \quad \searrow \\
\begin{array}{cc}
-A^2_{p_1} & +B^2_{p_1} \\
\perp_{p_1,p_2} & \perp_{p_1,p_{2'}}
\end{array}
\end{array}$$


Inference
Conclusion does not follow from the premises (*non sequitur*)
Conclusion does follow from the premises
Premises are not relevant to the conclusion (*non causa ut causa*)
Premises are relevant to the conclusion
Conclusion is not relevant to the premises (*quia*)
Conclusion is relevant to the premises (*propter quid*)


Propter quid
contraries
Non sequitur
subalterns
contradictories
subalterns
Quia
subcontraries
Non causa

	Axiom schemes or rules	*Propter quid*	*Quia*	*Non causa*	*Non sequitur*
(1)	$\phi \to \phi$	✓	✓	×	×
(2)	$((\phi \to \psi) \wedge (\psi \to \gamma)) \to (\phi \to (\psi \wedge \gamma))$	✓	✓	×	×
(3)	$\phi \to \psi, \phi \vdash \psi$	✓	✓	×	×
(4)	$\phi \to \psi \vdash (\psi \to \gamma) \to (\phi \to \gamma)$	✓	✓	×	×
(5)	$\psi \to \gamma \vdash (\phi \to \psi) \to (\phi \to \gamma)$	✓	✓	×	×
(6)	$\phi \to \neg\psi \vdash \psi \to \neg\phi$	✓	✓	×	×
(7)	$\phi \vdash \phi$	×	✓	✓	×
(8)	$\phi \to (\psi \to \phi)$	×	✓	✓	×
(9)	$\phi \to (\neg\phi \to \psi)$	×	✓	✓	×
(10)	$\phi \to (\psi \to \psi)$	×	✓	✓	×
(11)	$(\phi \wedge \psi) \to \phi$	×	✓	✓	×
(12)	$\phi \to (\phi \vee \psi)$	×	✓	✓	×
(13)	$\phi \to \psi, \psi \vdash \phi$	×	×	✓	✓
(14)	$\phi \to \psi, \neg\phi \vdash \neg\psi$	×	×	✓	✓
(15)	$\phi \vee \psi, \phi \vdash \neg\psi$	×	×	✓	✓
(16)	$\neg(\phi \wedge \psi), \neg\psi \vdash \phi$	×	×	✓	✓
(17)	$(\phi \to \psi), (\phi \to \gamma) \vdash (\psi \to \gamma)$	×	×	✓	✓
(18)	$(\phi \to \psi), (\psi \to \gamma) \vdash (\phi \wedge \gamma)$	×	×	✓	✓

So far so good. The issue is, however, that even if the previous method works consistently, it lacks philosophical warrant. In other words, we have the syntax, but we are still missing some philosophical semantics.

Given this situation, we now suggest what we call deliberative semantics in order to accommodate said method.

$$\pm\top^{i}_{f}$$

- What do the terms, the functors, the indexes and flags mean?
- What is the precise difference between a *quia* (*non causa*) and a *propter quid* (*non sequitur*) inference?
- And overall, how can we make sense of the four different types of tableaux obtained from using this proposal?

In order to answer these questions and, moreover, in order to develop some formal semantics, we will follow the next path.

1. First, we will try to explain that there are good reasons to reject *petitio* inferences as probative or relevant, even if they are truth-preserving.
2. This remark, that stems from the difference between a premise and a conclusion, will lead us to distinguish between *quia* and *propter quid* inferences in terms of their probative features.
3. These features, in turn, will be linked to a difference between premises and conclusions in terms of means-ends deliberation.
4. Finally, we will try to connect these distinctions with TFL's regular semantics.

1. So, first, that circular reasoning is not relevant or probative can be justified as follows. In *Pos. An.* 1, III, 72b25-32 Aristotle developed an interesting argument to reject *petitio* as a well-behaved, probative inference. A probative inference, according to Aristotle, is an inference in which the conclusion, although related to, has to be different from the premises (*Topics* 100a25-26, *De Sophisticis Elenchis* 165a1-2, *Prior Analytics* 24b19-20), but in a *petitio* the same statement works both as a premise and as a conclusion.

The issue is, then, that if a premise is prior to and better known than a conclusion (*Pos. An.* 1, I, 71a1-9 and *Pos. An.* 1, X, 76a32-36), then it would follow that the same statement is both prior to and subsequent to it, and also more known and less known, which is absurd. Thus, this argument would suggest that circular inferences, even if truth-preserving, cannot be probative.

2. This provides some reason to further distinguish between *quia* and *propter quid* inferences (*Pos. An.* 1, XIII, 78a22-30), for there is a difference between understanding *that* something is (which is what *quia* means) and understanding *why* something is (which is what *propter quid* means). For Aristotle, a probative or *propter quid* inference is one that proves the conclusion by proper reasons, namely, by reclaiming all the adequate links between terms, something that, by the way, does not occur with *petitio*, *ex falso* or *verum ad*.

This whole picture makes sense. Surely, we can prove anything using *petitio* or *ex falso*, and everything is a proof of a truth (*verum ad*), but such proofs are not causally relevant, for they do not reclaim all the appropriate links between terms. In particular, *petitio* does not reclaim terms from the premises; *ex falso* does not reclaim terms from the conclusion; and *verum ad* does not reclaim terms from the premises.

In a rather traditional fashion we would say in said inferences we have truth, but we lack knowledge of why we have such truth. And so, from this second argument we can conclude that, clearly, circular inferences (not to mention *ex falso* and *verum ad*) are not *propter quid* or probative, but only *quia* (*non causa*) or merely truth-preserving, and the reason why this is so is we have to properly distinguish premises from conclusions, which gives us more reasons to ground the use of the flags.

3. There is a general intuition that suggests some sort of simmetry between inference and deliberation in the sense that going from premises to conclusions is not only a truth oriented activity, but also a means-end ordered task (*Nic. Eth.* 1, I, 1094a1-3; *Nic. Eth.* 3, II, 1111b25-29). An argument in favor of this simmetry would go like this: means are to ends what premises are to conclusions, but means are not ends (and vice versa), so the reason we should syntactically distinguish premises from conclusions—for instance, using flags—is these elements of deliberation—and hence of inference or demonstration—are not semantically equivalent, and this fact, we think, provides some philosophical meaning and grounding to the use of the flags.

4. TFL semantics.

Term
- Expresses *a concept*
- Signifies *a property*
- Denotes *an individual*

Sentence
- Expresses *a proposition*
- Signifies *a fact*
- Denotes *a world*

Augmented semantics (informally).

Term { Expresses *a concept*
Signifies *a property*
Denotes *an individual*
Stands *in a position*

Sentence { Expresses *a proposition*
Signifies *a fact*
Denotes *a world*
Stands *in a position*

Augmented semantics (formally).

- $\mathfrak{D} = \langle \mathcal{T}, \mathcal{D}, \mathcal{F} \rangle$
  - $\mathcal{T} = \{\pm \mathsf{A}, \pm \mathsf{B}, \ldots\}$
  - $\mathcal{D} : \mathcal{T} \mapsto \mathbb{N}$
  - $\mathcal{F} : \mathcal{T} \mapsto \{p_i, c\}$, for $i \in \{1, 2, 3, \ldots\}$

Augmented semantics (results).

Theorem 1

A tableau is propter quid if and only if the corresponding inference is fully deliberative.

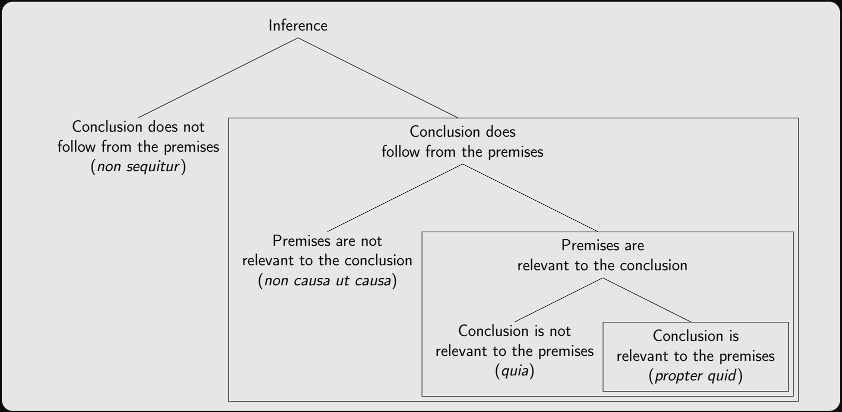
Theorem 2

A tableau is quia (non causa) if and only if the corresponding inference is partially deliberative.

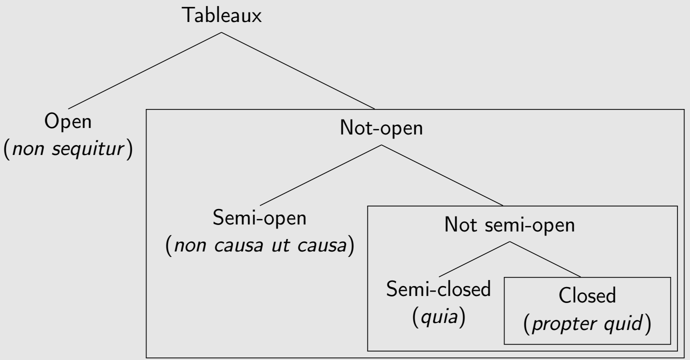
Theorem 3

A tableau is non sequitur if and only if the corresponding inference is non-deliberative.

Augmented semantics (visually).



Augmented semantics (visually).



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- *Objection 3. Current relevant logics are developed in terms of some intuitive semantics for implication, but what are the intuitions behind a term logic implication, if any?*

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Let us recap what we have done.

- We have seen that there is a particular sense of relevance captured by the traditional, Aristotelian logic,
- that TFL recovers some features of said logic,
- that we can develop a tableaux proof method for TFL, and that,
- by following an Aristotelian or syllogistic notion of relevance, we can offer some rules and some semantics that help move Sommers & Englebretsen's Term Functor Logic into the path of relevant logics.

We have proposed what we call deliberative semantics in order to accommodate a relevantoid version of Sommers and Englebretsen's Term Functor Logic. Our main results indicate that we can understand a formal, syntactical hierarchy of tableaux in terms of a semantics of deliberation that, if properly reviewed, could say a lot more of the difference between probative or causally relevant inferences and merely truth-preserving inferences. Or better yet, and simply put, our results suggest that relevant, probative inferences are pretty much, semantically, like well thought choices.

Thanks.

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